A Bayesian model for network flow data: an application to BikeMi trips

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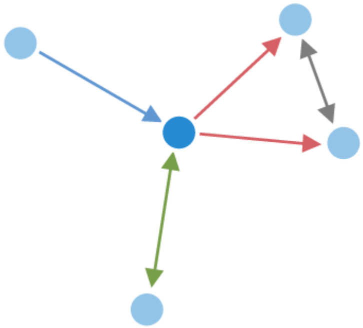
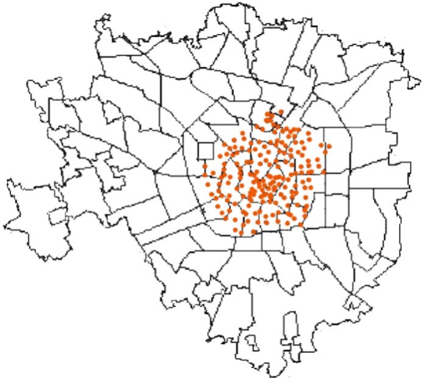
We are interested on making inference on a graph $\mathcal{G} = (V, E)$

- V: vertices
- $\mathrm{E} = \{(i, j)\}$ edges
- Quantity of interest Y_{ij}: flow on the edge (i, j).
  In our case, the flow is the number of trips
  from node i to node j



Dataset

- 263 BikeMi station
- **350093** trips between January 25th and March 6th 2016
- length of each trip



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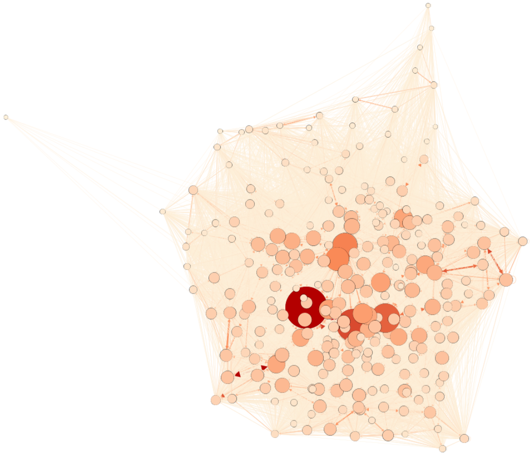
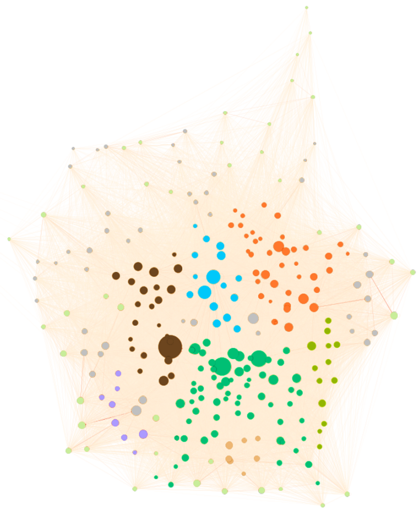
- $263^2 = \mathbf{69169}$ possible arcs!



Dataset

- 263 BikeMi station
- **350093** trips between January 25th and March 6th 2016
- length of each trip
- $263^2 = \mathbf{69169}$ possible arcs!
- DBSCAN clustering on the geolocation of the stations
- 67 nodes in the reduced network



The Bayesian Model

We consider the following Zero-inflated Poisson mixture regression model

$$p(Y_{ij} = y)|\theta, \boldsymbol{\mu}_{ij}, \boldsymbol{\lambda} \overset{\text{ind}}{\sim} \begin{cases} \theta + (1-\theta)\text{PM}(0|\boldsymbol{\mu}_{ij}, \boldsymbol{\lambda}) & \text{if } y = 0 \quad i, j = 1, \ldots, 67 \\ (1-\theta)\text{PM}(Y_{ij}|\boldsymbol{\mu}_{ij}, \boldsymbol{\lambda}) & \text{if } y = 1, 2, 3, \ldots \end{cases}$$

$$\text{PM}(\boldsymbol{\mu}, \boldsymbol{\lambda}) = \text{PM}\left((\mu_1, \ldots, \mu_K), (\lambda_1, \ldots, \lambda_K)\right) = \sum_{i=1}^{K} \lambda_i \text{Poi}(\mu_i)$$

$$\log \mu_{ijk} = \boldsymbol{\beta}_k \boldsymbol{x}_{ij}$$

For each arc, the covariates $\boldsymbol{x}_{ij}$ are $(1, S_i \times T_j, d_{ij})$

- S_i: "sourceness" of starting node (outer degree)
- T_j: "targetness" of destination node (inner degree)
- d_{ij}: geographical distance

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$$\boldsymbol{\beta}_k \overset{\text{iid}}{\sim} \mathcal{N}_3(\boldsymbol{m}, \Sigma) \quad k = 1, \dots, K$$
$$\boldsymbol{\lambda} \sim \text{Dirichlet}(\boldsymbol{\alpha})$$
$$\theta \sim \mathcal{U}(0, 1)$$

$$K = 4, \;\; \boldsymbol{m} = (7, 0, 0), \;\; \Sigma = \text{diag}(2, 1.5, 1.5), \;\; \boldsymbol{\alpha} = (2, 2, 2, 2)$$

Posterior Inference

MCMC simulation was performed using Stan software.

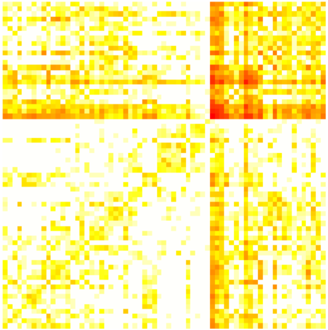


Figure: Observed flow

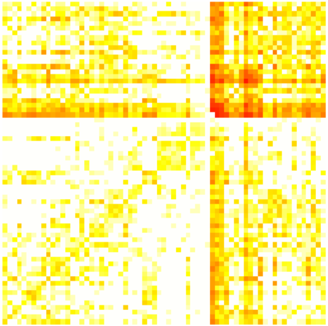
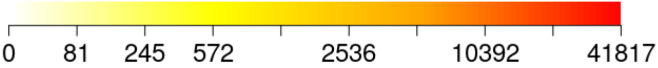


Figure: Predicted flow



The cluster estimate was found minimizing Binder's loss function.

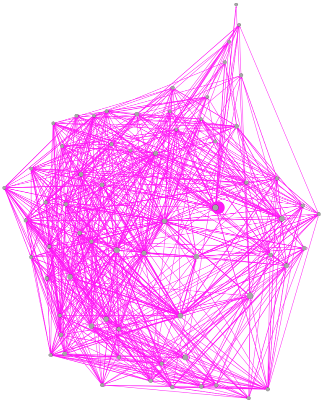


Figure: Estimated cluster 1

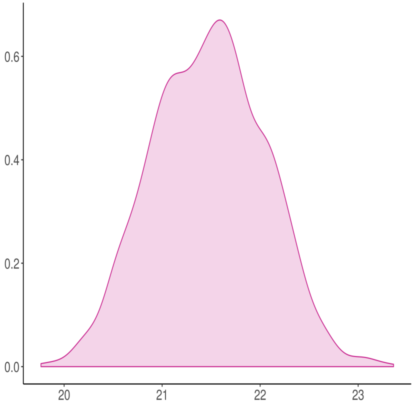


Figure: Posterior mean flow

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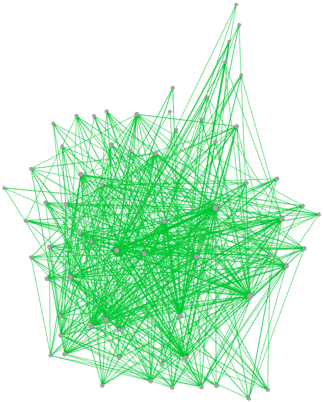
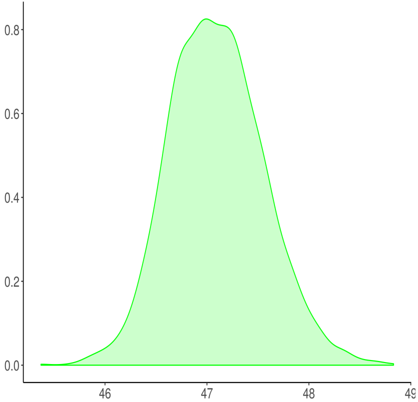


Figure: Estimated cluster 2



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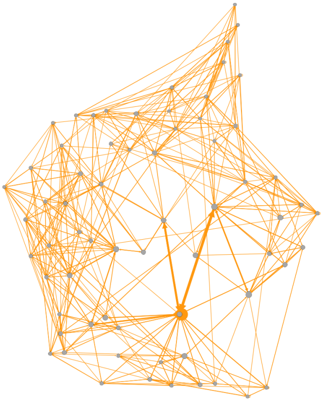
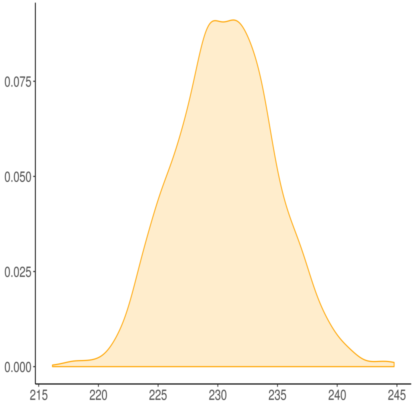


Figure: Estimated cluster 3



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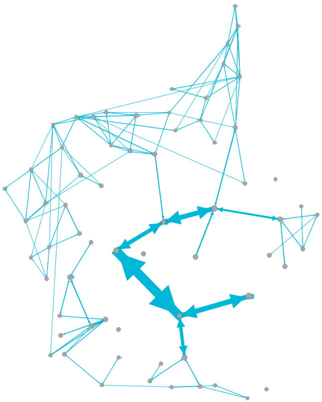
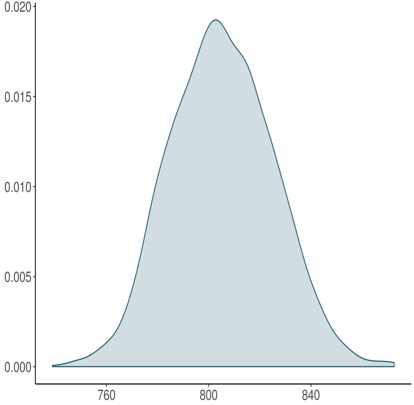


Figure: Estimated cluster 4



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Comparison

We assess the predictive performances of our model against other possible Bayesian "competitors".

- `Reg0infl`

$$Y_{ij}|\theta, \boldsymbol{\mu}_{ij}, \boldsymbol{\lambda} \overset{\text{ind}}{\sim} \begin{cases} \theta + (1-\theta)\text{PM}(0|\boldsymbol{\mu}_{ij}, \boldsymbol{\lambda}) & \text{if } Y_{ij} = 0 \\ (1-\theta)\text{PM}(Y_{ij}|\boldsymbol{\mu}_{ij}, \boldsymbol{\lambda}) & \text{if } Y_{ij} > 0 \end{cases}$$

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- `0infl`
- `Reg`

Model	MSE	LOO-ELPD	LPML
`Reg0infl`	**1,518.3**	**-14,270.5**	**-11,237.9**
`Reg`	1,685.4	-18,347.7	-18,341.7
`0infl`	300,373.4	-84,862.0	-103,089.3

Table: Predictive performances comparison.

Conclusions

- Presented a class of full Bayesian models to analyze the mobility of BikeMi
- Zero-inflated Poisson mixture regression models captures both the topology of the network and different range of behaviours in the flows
- Incorporate in the model edge-specific covariates

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Future developments

- Different clustering
- Considering the whole network
- Node specific covariates such as proximity to points of interest

References

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